Vapor Ingestion During Outflow

Prediction of the amount of liquid that remains in a tank when vapor ingestion occurs during discharge under low-g conditions is important for designing propellant transfer systems with minimum residual. Since surface displacements are large when vapor ingestion occurs, satisfactory prediction of liquid residual has not been possible with linearized, inviscid, free-surface-flow techniques. The MAC method utilizes the full Navier-Stokes equations, and as such has the capability for handling nonlinear effects.

Figure 3 shows some results for a 10-ft-diam tank, which had a 1-ft-diam drain in the center and was filled initially to a depth of 2.5 ft. A steady gravity level of $10^{-3}g_e$ and a flat initial liquid-vapor interface were considered. A 0.5-ft^2 calculation grid was utilized. The computing time increment was 0.04-sec, and the real-time duration was 6.0 sec. The computer time required was 6.5 min. The outflow velocity imposed as a boundary condition increased linearly from 0 to 5 fps in 1 sec and then remained constant. This outflow velocity was selected based on data from Ref. 2 as the value which will cause vapor ingestion to occur at a liquid-height/tank-diameter ratio (h/D) of 0.235. The outflow Froude number (Fr), as defined in Ref. 2, is 0.0781.

Calculated results in Fig. 3b show vapor ingestion initiating at 5.84 sec with $h/D = 0.23$, in close agreement with data of Ref. 2. Subsequent calculations made with larger initial liquid heights of 3.0 ft and 3.5 ft showed vapor ingestion at $h/D = 0.25$. Calculated h/D for a second, larger $Fr = 0.447$ compared similarly with data of Ref. 2.

References

[1] Welch, J. E. et al., "The MAC Method," LA-3425, March 1966, Los Alamos Scientific Lab., Los Alamos, N. Mex.

[2] Gluck, D. F. et al., "Distortion of a Free Surface During Tank Discharge," *Journal of Spacecraft and Rockets*, Vol. 3, No. 11, Nov. 1966, pp. 1691–1692.

Laminar Wall Jet with Blowing or Suction

SHOICHIRO FUKUSAKO,* MASARU KIYA,† AND MIKIO ARIE‡
Hokkaido University, Sapporo, Japan

Introduction

ALTHOUGH the effects of magnetic fields, suction or blowing, etc. on the boundary-layer flow along a solid surface have been extensively studied from the practical point of view of the boundary-layer control, very few studies are made concerning the flow properties and heat-transfer characteristics of a wall jet spreading over a permeable plane surface in otherwise stationary surroundings. The problem of laminar wall jet of this category was extensively studied by Fox and Steiger[1] by means of similar solutions. Their method of solution reduces to an eigenvalue problem and the distribution of suction and blowing velocities must be necessarily specified according to the eigenvalues. In fact, the normal velocity at the surface must be proportional to $x^{1/2(2\alpha-1)/(\alpha-1)}$, where x is a dimensionless coordinate along the surface and α is an eigenvalue. Although they state that an infinite number of similar solutions with each eigenvalue permit one to obtain a wide variety of distributions of normal velocity at the surface, we feel it worthwhile to construct a solution of more general applicability. In this Note suction and blowing velocities are assumed to be proportional to x^n, but, unlike Fox and Steiger's case, n may take any constant value. The solution is obtained as a perturbation from the laminar wall jet over an impermeable surface the solution of which was first obtained by Glauert.[2] Specific considerations are given to the case of wall jet over an isothermal permeable surface with uniform suction and blowing, the case of which is not included in Fox and Steiger's solution.

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* Graduate Student, Department of Mechanical Engineering.
† Assistant Professor, Department of Mechanical Engineering.
‡ Professor, Department of Mechanical Engineering.

Analysis

The wall jet can be treated within the framework of the boundary-layer theory (Glauert[2]). In the following analysis, all velocities are referred to a characteristic velocity U and all lengths to ν/U. When the two-dimensional flow of an incompressible viscous fluid is steady and laminar, the boundary-layer equations become

$$\partial u/\partial x + \partial v/\partial y = 0 \tag{1}$$

$$u\partial u/\partial x + v\partial u/\partial y = \partial^2 u/\partial y^2 \tag{2}$$

$$u\partial T/\partial x + v\partial T/\partial y = (1/P_r)\,\partial^2 T/\partial y^2 \tag{3}$$

where x is the streamwise distance measured along the surface from an appropriate origin, y is the distance normal to the surface, u, v are the corresponding velocity components and T is the temperature. P_r is the Prandtl number. The isothermal surface temperature is T_w and the temperature of the surrounding fluid is T_∞. The boundary conditions are

$$y = 0:\quad u = 0,\ v = v_w x^n,\ \theta = 1 \tag{4}$$

$$y = \infty:\quad u = 0,\ \theta = 0 \tag{5}$$

where v_w is a constant and θ means $(T - T_\infty)/(T_w - T_\infty)$. Uv_wx^n is the dimensional blowing or suction velocity; negative v_w implies suction and positive v_w blowing.

The continuity equation, Eq. (1), is automatically satisfied if the stream function ψ is introduced by the usual definition

$$u = \partial\psi/\partial y,\ v = -\partial\psi/\partial x$$

Here, we introduce the new variables

$$\xi = v_w x^{[(3/4)+n]},\ \eta = \tfrac{1}{2}yx^{-(3/4)}$$

and put the stream function and the temperature into the form

$$\psi(x,y) = 2x^{(1/4)}F(\xi,\eta)$$

$$\theta(x,y) = h_0(\eta) + \xi h_1(\eta) + \ldots \tag{6}$$

where

$$F(\xi,\eta) = f_0(\eta) + \xi f_1(\eta) + \ldots$$

The velocity components can now be expressed in terms of ξ and η

$$u = x^{-(1/2)}\partial F/\partial\eta \tag{7a}$$

$$v = -\frac{1}{2}x^{-(3/4)}\left[F - 3\eta\frac{\partial F}{\partial\eta} + (3+4n)\xi\frac{\partial F}{\partial\xi}\right] \tag{7b}$$

With Eqs. (6, 7a, and 7b), the momentum and energy equations can be written in terms of the new variables. Then, in each of the resulting equations, the terms are gathered according to the powers of ξ which multiply them. In this way, one obtains a set of ordinary differential equations for the functions f_0, h_0, f_1, h_1, . . .

$$f'''_0 + f_0f''_0 + 2(f'_0)^2 = 0 \tag{8a}$$

$$h''_0 + P_rf_0h'_0 = 0 \tag{8b}$$

$$f'''_1 + f_0f''_1 + 4(n+1)f_1f''_0 + (1-4n)f'_0f'_1 = 0 \tag{8c}$$

$$h''_1 + P_r[f_0h_1 + 4(n+1)f_1h'_0 - (4n+3)f'_0h_1] = 0 \tag{8d}$$

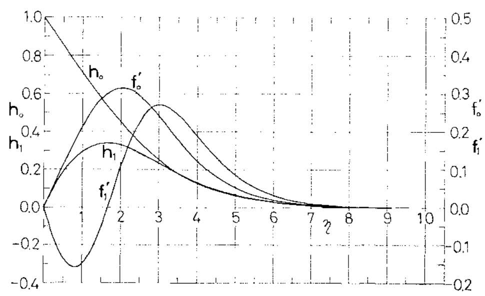

Fig. 1 The functions f'_0, f'_1, h_0, and h_1.

where the primes denote differentiation with respect to η. It should be noted that Eq. (8a) coincides with the governing equation for the laminar wall jet over an impermeable surface, the solution of which is plotted by Glauert.[2] Once f_0 is known, it is an easy matter to solve Eq. (8b) numerically. It thus remains to solve Eqs. (8c) and (8d), and the results will provide the first-order effects of the blowing or suction on the wall jet and the heat transfer. Corresponding to the boundary conditions given by Eqs. (4) and (5), the functions f and h take the following boundary values:

$$\eta = 0: \quad f_0 = f'_0 = 0, h_0 = 1$$

$$f_1 = -1/[2(n+1)], f'_1 = h_1 = 0$$

$$\eta = \infty: \quad f'_0 = h_0 = 0, f'_1 = h_1 = 0$$

By utilizing the Runge-Kutta-Gill method, the solutions of Eqs. (8c) and (8d) were obtained numerically for $P_r = 0.72$ and $n = 0$ on HIPAC 103 electronic digital computer installed at the Computer Center of Hokkaido University. The calculated results are tabulated in Table 1, together with the values of $f''_0(0)$ and $h'_0(0)$ that were used as the input data for Eqs. (8c) and (8d). The information contained in Table 1 will be used in the skin-friction and heat-transfer calculations which follow. Additionally, the functions f'_0, f'_1 and h_0, h_1, respectively, related to the velocity and temperature profiles, are plotted in Fig. 1.

Skin Friction and Heat Transfer

The skin friction τ may be calculated from $\tau(x) = (\partial u/\partial y)_{y=0}$. Introducing the variables ξ and η, the skin friction takes the form

$$\tau(x)/\tau_0(x) = 1 + [f''_1(0)/f''_0(0)]\xi + \ldots$$
$$= 1 - 1.72494(v_w x^{(3/4)}) + \ldots \quad (9)$$

where $\tau_0(x)$ is the skin friction when suction or blowing is absent. Equation (9) provides the expected result that blowing decreases the skin friction, while suction increases the skin friction.

The local heat transfer $q(x)$ passing per unit area from the surface to the fluid can be evaluated by Fourier's law. In the same way as the skin friction, one obtains the expression

$$q(x)/q_0(x) = 1 + [h'_1(0)/h'_0(0)]\xi + \ldots$$
$$= 1 - 1.55618(v_w x^{(3/4)}) + \ldots \quad (10)$$

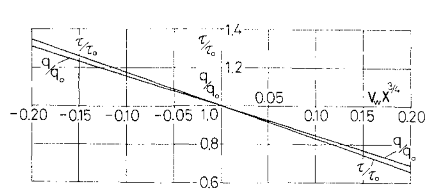

Fig. 2 Effect of blowing and suction on local skin friction and local heat transfer.

Table 1 Velocity and temperature derivatives

$f''_0(0)$	$f''_1(0)$	$h'_0(0)$	$h'_1(0)$
0.22222	−0.38332	−0.28676	0.44625

in which $q_0(x)$ represents the heat-transfer rate for the case of an isothermal, impermeable wall. Corresponding to the skin-friction case, blowing decreases the heat transfer, while suction increases the heat transfer. To facilitate interpretation of the results, Eqs. (9) and (10) have been plotted in Fig. 2.

Discussion of Results

To obtain perspective as to the magnitude of the blowing and suction velocities involved, one may use as a reference quantity the maximum velocity $u_{\max}$ corresponding to an isothermal impermeable wall, as was done by Sparrow and Cess[3] in the case of free convection due to a heated vertical flat plate with blowing and suction at the surface. From Glauert,[2] $u_{\max}$ is given by

$$u_{\max} = 0.315x^{-(1/2)}$$

By the use of this relation and the expression for ξ with $n = 0$, it is easy to show that

$$v_w/u_{\max} = 3.18x^{-(1/4)}\xi$$

Suppose one considers an abscissa value of ± 0.1, which corresponding to a 17% change in skin friction and a 16% change in heat transfer, respectively. Then, $v_w/u_{\max} = 0.318x^{-1/4}$. For a current length of 10^4, $v_w/u_{\max} = 0.0318$; while when $x = 10^6$, $v_w/u_{\max} = 0.0100$. It is thus demonstrated that very small blowing and suction velocities can significantly effect the skin friction and heat transfer.

References

[1] Fox, H. and Steiger, M. H., "Some Mass Transfer Effects on the Wall Jet," *Journal of Fluid Mechanics,* Vol. 15, 1963, pp. 597–609.

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Spin Decay of Explorer XX

P. C. Hughes* and D. B. Cherchas†
University of Toronto, Toronto, Canada

The Explorer XX topside sounder satellite was spin-stabilized about its major inertia axis at about 1.5 rpm, but experienced in a subsequent loss of spin. The mechanism for this loss of angular momentum has been described[1] in terms of an interaction with the solar radiation field. Originally, the incentive for the investigation was the Canadian Alouette I satellite which is similar in many respects to the Explorer XX.

The theory was formulated in terms of continuous sunlight and it also presumed that the solar vector and the satellite spin vector were perpendicular. The validity of the approach was then established on the basis of the correct shape of the spin history curve predicted by this theory, and the reasonable

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* Assistant Professor, Institute for Aerospace Studies.
† Graduate Student, Institute for Aerospace Studies.